AP Calculus AB
PRACTICE Sheet 6

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Write all subsets of $\mathbb{R}$ as the union of disjoint intervals or finite sets.

Problem 1. (Domain) Let $f(x) = \sqrt{9 - x^2}$. Find the domain and range of f.

Problem 2. (Limits) Let $f(x) = \dfrac{x^3 - 7x^2 - x - 56}{x^2 - 9x + 8}$. Find $\lim_{x \to 8} f(x)$.

Problem 3. (Polynomials) Consider the polynomial function

$$g(x) = 12x - x^3.$$

Find the zeros and intercepts of g, and sketch its graph.

Problem 4. (Domain of Composition) Let $f(x) = \sqrt{x + 3}$ and $g(x) = \dfrac{1}{x - 5}$. Find $\mathrm{dom}(g \circ f)$ and $\mathrm{dom}(f \circ g)$.

Problem 5. (Derivatives) Let $f(x) = x^5 - 2x^4 + 3x^3 - 5x^2 + 7x - 11$. Find $f'(x)$.

Problem 6. (Derivatives) Let $f(x) = \dfrac{3}{x} + 5\sqrt{x} + 7\sin x + 11e^x$. Find $f'(x)$.

Problem 7. (Domain) Let $f(x) = \sqrt{x^2 - 2x - 15}$. Find $\mathrm{dom}(f)$.

Problem 8. (Range) Let $g(x) = x^2 - 14x + 100$. Find $\mathrm{range}(g)$.

Problem 9. (Limits) Let $f(x) = \dfrac{x^3 - 16x^2 + 57x - 22}{x^2 - 4x - 77}$. Find $\lim_{x \to 11} f(x)$.

Problem 10. (Continuity) Let

$$f(x) = \begin{cases} 25 - x^2 & \text{for } x \leq 3 \\ (x - c)^2 & \text{for } x > 3 \end{cases}$$

Suppose f is continuous. Find all possible values for c.

Problem 11. (Tangents) Let $g(x) = x^3 - 7x + 3$. Find the equation of the line tangent to the graph of g at $x = 2$.

Problem 12. (Piecewise Limits) Let

$$f(x) = \begin{cases} \sqrt{x + 3} & \text{for } x < 6 \\ 0 & \text{for } x = 6 \\ 25 - x^2 & \text{for } x > 6 \end{cases}$$

Find $\lim_{x \to 6^-} f(x)$ and $\lim_{x \to 6^+} f(x)$.

Problem 13. (Piecewise Continuity) Let

$$f(x) = \begin{cases} x^2 + k & \text{for } x < 2 \\ x + 5 & \text{for } x \geq 2 \end{cases}$$

Find the value for k such that $\lim_{x \to 2} f(x)$ exists.

Problem 14. (Theory)

Reproduce the proof given in class that $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$, and use it to show that $\dfrac{d}{d\theta}\sin\theta = \cos\theta$.

Problem 15. (Theory)

Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. We have discussed why $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \dfrac{f(x-h) - f(x)}{-h}$. Use this fact, and the definitions of even and odd functions, to show that if f is an odd function, then f' is an even function.

Problem 16. (Wrapping Function) Let $W : \mathbb{R} \to \mathbb{R}^2$ be the wrapping function.

(a) Find $W\left(\dfrac{79\pi}{6}\right)$.

(b) Let $F(t) = W(2\pi t)$. Find $F\left(\dfrac{51}{12}\right)$.

(c) Find all $t \in \mathbb{R}$ such that $W(t) = (x, y)$ and $x = y$.

Problem 17. (Rational Limits) Compute the limit.

(a) $\lim_{x \to 2} x^2 - 3x - 40$

(b) $\lim_{x \to 2} \dfrac{x^2 - 3x - 40}{x - 5}$

(c) $\lim_{x \to 5} \dfrac{x^2 - 3x - 40}{x - 5}$

(d) $\lim_{x \to 8} \dfrac{x^2 - 3x - 40}{x - 8}$

(e) $\lim_{x \to \infty} \dfrac{x^2 - 3x - 40}{x^2 - 8}$

Problem 18. (Math Facts)
Let a and C be constants, and let u and v be functions of x.

(1) $\dfrac{d}{dx}C =$ ________________________ (constant rule)

(2) $\dfrac{d}{dx}(u+v) =$ ________________________ (sum rule)

(3) $\dfrac{d}{dx}(au) =$ ________________________ (constant multiple rule)

(4) $\dfrac{d}{dx}x^n =$ ________________________ (power rule)

(5) $\dfrac{d}{dx}\sqrt{x} =$ ________________________

(6) $\dfrac{d}{dx}\dfrac{1}{x} =$ ________________________

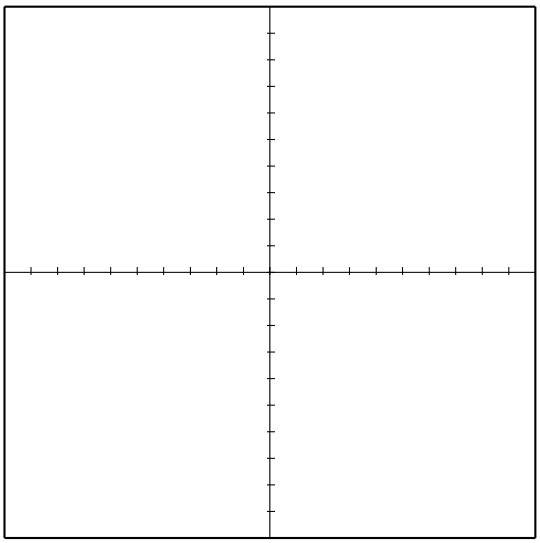
(7) $\dfrac{d}{dx}\sin(x) =$ ________________________

(8) $\dfrac{d}{dx}\cos(x) =$ ________________________

(9) $\dfrac{d}{dx}e^x =$ ________________________

(10) $\dfrac{d}{dx}\ln x =$ ________________________

Problem 19. (Graphing Polynomials) Consider the polynomial function $f(x) = x^4 - 5x^2 + 4$. Find its degree, leading coefficient, constant coefficient, zeros, and end behavior. Find the y-intercept and x-intercepts. Graph the function and label these points.



Polynomial: $f(x) = x^4 - 5x^2 + 4$

Degree:

Leading Coefficient:

Constant Coefficient:

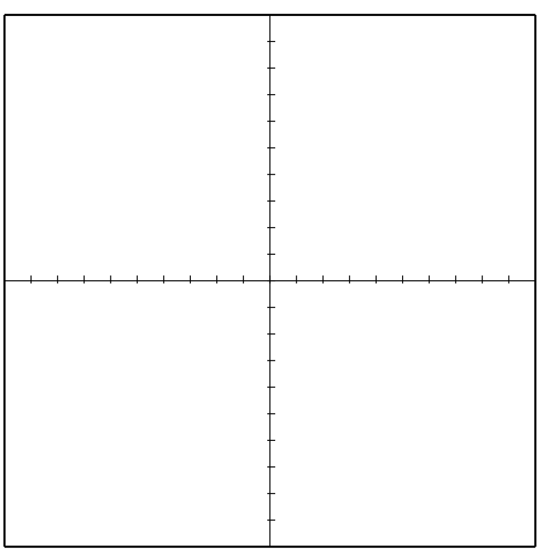
Zeros:

y-intercept:

x-intercepts:

End Behavior:

Problem 20. (Graphing Rational Functions) Consider the rational function $f(x) = \dfrac{3x^2 - 12}{x^2 - 9}$. Find its degree, zeros, and poles. Find its intercepts and asymptotes. Graph the function and label these features.



Rational Function: $f(x) = \dfrac{3x^2 - 12}{x^2 - 9}$

Degree:

Zeros:

Poles:

y-intercept:

x-intercepts:

Vertical Asymptotes:

Polynomial Asymptote: